Asset Selection based on Level-Dependent Utility

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Abstract: Portfolio analysis is one of the classic problems in economics. It is about solving how investors maximize their investment returns or minimize investment risks. Based on the level-dependent utility model, this paper constructs a portfolio model with utility maximization. The differential evolution method is used to solve the model, and the constraint conditions in the model are processed by the penalty function method. The rationality and feasibility of the model are verified by empirical analysis.

1. Introduction

In economics, one of the more discussed topics is the choice of portfolio. Modern Portfolio Theory (MPT) was proposed by Markowitz in 1952. He published a paper entitled "Selection of Portfolios" in the March 1952 issue of Financial Journal to determine the set of minimum variance portfolios. The ideas and methods have created a precedent for the overall management of investment and laid the foundation for the development of portfolio investment theory. The central issue of this theory is how people choose the combination of benefits and risks in securities investment decisions. That is, maximizing the expected utility at a given level of expected risk, or minimizing the expected risk at a given desired level of return.

Based on the utility theory proposed by Daniel Bemoulli, Von Neumann and Morgenstern first proposed the expected utility theory (EU) of investors' decision under risk. It is believed that the ultimate utility function of investors should be expressed as the product of objective probability and value, that is, investors are rational and risk-averse, and the criterion of investment decision-making is maximization of utility. But the EU theory assumes that investors are completely rational, which is contrary to reality. In view of the shortcomings of the theory of expected utility, many scholars added the relevant theories of psychology when conducting economic research, and supported the actual data as the unexpected utility theory to solve the decision problem under uncertain conditions. Quiggin proposed a level-dependent utility theory (RDU) that belongs to the category of unexpected utility theory. The theory is great innovation significance in distinguishing between decision weight and probability weight. The choice of cumulative probability weight function can explain Allais paradox well, overcoming the limitation that traditional expectation utility theory can not fully describe the attitude and degree of economic man's uncertainty risk.

Based on the actual situation in China, this paper will use the differential evolution method to obtain the optimal investment portfolio by establishing a level-dependent portfolio model.

2. Level Dependent Utility Model

In the RDU model, as with the basic expected utility theory, the utility function is generally assumed to be concave or linear. However, the weight function $\pi(p)$ as a transfer function of probability can no longer be purely linear, concave or convex, but can be constructed into a mixed form, which can be changed according to different decision-making psychology of the decision maker.

Suppose random variable in the set $\{x_1, x_2, \cdots, x_S\}$, and specify that $x_1 < x_2 < \cdots < x_S$. Obey the probability distribution $\Pr\{X = x_s\} = p_s$, $s = 1, 2, \cdots, S$, and satisfied $p_s \geq 0$, $\sum p_s = 1$. Then the level dependent utility can be expressed as:

$$RDEU(r) = \sum_{s=1}^{S} u(r_s)\pi(p_s)$$

$$\begin{cases} \pi(p_s) = \Phi(\sum_{j=s}^{S} p_s) - \Phi(\sum_{j=s+1}^{S} p_s), \forall s = 1, \cdots, S-1 \\ \pi(p_S) = \Phi(p_S) \end{cases}$$

It can be seen that the model consists of two parts. One part is the utility function $u(r)$, this article uses the exponential utility function to substitute the solution: $u(r) = \frac{1-e^{-\alpha r}}{1-e^{-\alpha}}$, ($\alpha \geq 0$).The other part is the weight function $\pi(p)$. In the level-dependent utility function, $\pi(p)$ is no longer the probability of the result, but the subjective factor of the decision maker, namely the emotion factor $\Phi(x)$. $\Phi(x)$ is a monotonically increasing function whose domain is [0,1], continuous and differentiable in the interval (0,1). At the same time $\Phi(0) = 0$, $\Phi(1) = 1$. This article defines the emotional function as $\Phi(x) = x^{\beta}$.When 0<β<1, it means that the decision maker is optimistic; when β=1, it means no emotion; when β>1, means pessimistic.

3. Portfolio Model Based on Rank-Dependent Utility

3.1 Model Representation

In the portfolio model, it is generally assumed that *N* is the number of assets, *S* is the number of situations. p_s is the probability of occurrence of *s*, r_{is} is the rate of return of asset *i* at *s*, and $\omega_i \geq 0$ is the weight of asset *i* in the portfolio. A portfolio can be expressed as $x_i = (\omega_1, \ldots, \omega_N)$, $\sum_{i=1}^{N} \omega_i = 1$. This paper uses the expected utility maximization to find the optimal portfolio, so the level-dependent utility portfolio model can be expressed as:

$$\max RDEU(x) = \sum_{s=1}^{S} u(\sum_{i=1}^{N} r_s \omega_i) \cdot \pi(p_s)$$

$$\begin{cases} \sum_{i=1}^{N} \omega_i = 1 \\ \omega_i \geq 0 \end{cases}$$

3.2 Algorithm Description

The Differential Evolution (DE) was first proposed by Storn and Price in 1995. It is a heuristic algorithm with a simple principle and few controlled parameters. Therefore, this paper uses the differential evolution algorithm to solve the above portfolio.

Firstly, initialize the population. P^2 individuals are randomly generated, each containing *N* vectors. Secondly, Variation. By randomly selecting 3 individuals x_a, x_b, x_c, and ensure $x_a \neq x_b \neq x_c \neq x_i$, generating variation vector $h_i = x_a + F \cdot (x_b - x_c)$. $F \in [0, 2]$ is scaling factor. Thirdly, cross. x_i and the variation vectors h_i are crossed according to the conditions of the following formula:

$$\tilde{x}_i = \begin{cases} h_i, R = j \text{ or } rand(0,1) \leq CR. \\ x_i, else. \end{cases}$$

Among them, $R \in \{1, \ldots, N\}$ is a randomly generated random number. $CR \in [0,1]$ is crossing probability. Finally, By calculating $RDEU(\tilde{x}_i)$ and $RDEU(x_i)$, select the largest individual to become the new individual $\tilde{y}_i$ and enter the next iteration.

3.3 Constraint Handling

Because of the constraints in the model, this paper uses the outer point penalty function to process. Let $g(\omega)=\sum_{i=1}^{N}\omega_i-1$, then the penalty function can be expressed as: $P_{c_k}(\omega)=c_k[\max(g(\omega),0)]^2+\frac{c_k}{2}\sum_{j=1}^{q}\omega_i^2$, where $\{c_k\}$ is a penalty parameter column. Then the transformed unconstrained objective function is expressed as $F_{c_k}(x)=REDU(x)+P_{c_k}(\omega)$. By giving the error of the test termination condition $\varepsilon>0$, let $k=1,2,\cdots$, iterate to find the best P_{c_k}.

3.4 Pseudo-code

```
Input:Population: P^2 ;Dimension: N ;Generation: G
Output:the best vector - y_i
Initial population x_i , i=1,...,P^2
while 1
ck
while g<=G
    for i=1: P^2
        choose 3 random vectors x_a ≠ x_b ≠ x_c ≠ x_i
        for j=1:N
              Pick u_j ~ U(0,1)
              if u_j < CR or j = R
                      x̃_ij = x_aj + F·(x_bj − x_cj)
              else
                      x̃_ij = x_ij
              end
        end
        if F_ck(x̃_i) > F_ck(x_i)
              ỹ_i = x̃_i
        else
              ỹ_i = x_i
        end
    g=g+1
  end
  if P_ck < ε
      break
  else
      k=k+1
  end
end
```

4. Empirical Analysis

In this paper, the program for the above model is written in Matlab. he data uses the 10 constituent stocks with the highest weight in the CSI 300 Index, and the daily closing price p_{is} is selected from August 1, 2018 to August 31, 2018. Here, the logarithm of the price ratio is used as the daily rate of return, $r_i = \ln\left(\frac{p_{is}}{p_{is-1}}\right), i = 1, \ldots, N$, $s = 1, \ldots, S$ .The sum of the yields of the portfolios at each moment is first sorted before running. Set the scaling factor F = 0.8, the crossover probability CR = 0.5, the number of iterations G = 100, P = 20. The results are shown in Table 1.

Table 1. Optimal Portfolio of Tests

secID	**600016**	**651**	**600036**	**601166**	**333**	**601318**	**600887**	**601328**	**600519**	**600276**
optimistic	0.0541	0.0293	0.0588	0.0259	0.1044	0.5212	0.0359	0.0154	0.0208	0.2286
pessimistic	0.6211	0.0021	0.0036	0.002	0.0194	0.2729	0.0005	0.005	0.0004	0.0053

5. Summary

In this paper, the differential utility model with penalty function is used to solve the level-dependent utility model, and the feasibility of this method in portfolio research is verified by empirical analysis. It can be found that optimistic and pessimistic decision makers have different investment portfolio rights. Therefore, the level-dependent utility model can be used to make investment decisions based on different attitudes.

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